The Implementation of Two Channel IIR Quadrature Mirror Filter Bank Based on Residue Arithmetic

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***Abstract*—Design of two-channel quadrature mirror filter (QMF) bank constructed by real infinite impulse response (IIR) digital all-pass filter is considered. Residue number system offers parallel processing and digital hardware implementation for binary operation of addition, subtraction and multiplication. There is no carry propagation between these operations. Hence, devices operating in this principle inherit properties of high speed and low power consumption. However, this properties makes overflow detection difficult during any operation. This paper discusses the application of RNS in designing the infinite impulse response (IIR) filter.**

***Keywords*—Quadrature Mirror Filter (QMF) bank, RNS scaler, all-pass filter, FFT, Chinese Remainder Theorem (CRT).**

I. Introduction

For many communication and signal processing systems, quadrature mirror filter banks have been widely used to achieve the goals of subband coding [1]. Generally, QMF bank is applied to decompose a signal into two subbands and decimate the subband signals in the analysis system by an integer, equal to the number of subbands. Also, two-channel QMF banks are used for constructing the M-channel QMF banks based on a tree structure.

The application of Residue Number System (RNS) in digital signal processing has been studied in paper [3]. The carry free nature and parallelism properties of this number system make it convenient for digital signal processing where most of the computations are additions and multiplications.

In comparison to Finite Impulse Response (FIR) filter design, Infinite Impulse Response (IIR) filters are more representative in classical signal processing and are prevalent in digital signal processing applications. They are generally smaller in process of implementation but require scaling of intermediate values during the signal processing.

In this paper, new design and implementation of an RNS-based two channel IIR quadrature mirror filter, is presented. The requirements for the filter coefficient word length are studied. This architecture is compared to the equivalent system based on two's complement arithmetic, showing considerable improvement in performance.

II. Quadrature Mirror Filter Bank

Basically, the Quadrature Mirror Filter (QMF) is a parallel combination of a High Pass Filter (HPF) and a Low Pass Filter (LPF), and it performs the action of frequency subdivision by splitting the signal spectrum into two spectra. It is a two channel subband coding filter bank with complementary frequency responses. It consists of two sections: analysis and synthesis section. The analysis subband filters have transfer functions $H_0(z)$ and $H_1(z)$ as shown in equation (2).

A. Digital All-Pass Based QMF Bank

Two-channel QMF bank, with analysis structure shown by Fig. 1, is considered, where $A_1(z^2)$ and $A_2(z^2)$ are two real

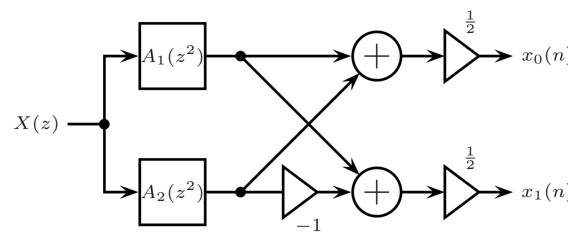

Figure 1. The analysis system of two-channel QMF bank.

IIR digital all-pass filters. The lowpass $H_0(z)$ and the highpass $H_1(z)$ filters of the analysis section are given by

$$H_0(z) = \frac{1}{2}[A_1(z^2) + A_2(z^2)], \quad (1)$$

and

$$H_1(z) = \frac{1}{2}[A_1(z^2) - A_2(z^2)], \quad (2)$$

where $A_1(z)$ and $A_2(z)$ are real stable all-pass filters of orders M and N, respectively. This contribution concentrates on designing lowpass filters. In this case, $M = N - 1$ or $M = N + 1$ so that the overall degrees $(M + N)$ of $H_0(z)$ and $H_1(z)$ are odd. This pair is a power complementary filter pair since it satisfies following relation:

$$|H_0(e^{j\omega})|^2 + |H_1(e^{j\omega})|^2 = 1. \quad (3)$$

Therefore, a complementary filter pair (e.g., a lowpass/

highpass) can be implemented with a cost of a single filter. Moreover, $H_0(z)$ and $H_1(z)$ satisfy the all-pass complementary and power complementary properties. They are termed the doubly-complementary (DC) filter pair [4].

B. Approximation

The odd degree lowpass filters, constructed as parallel connection of two allpass filters, can be easily designed directly in z domain without any reference to the continuous-time prototype network. Required parameters for allpass subfilters construction, are pole locations of the transfer function. Information about the numerator is not needed.

The magnitude-squared transfer function of the N-th degree IIR digital filter can be specified by

$$|H_0(x)|^2 = \frac{1}{1+K_N^2(x)}, \tag{4}$$

where $K_N(x)$ is the characteristic function and x is frequency variable. At the passband edge it is $K_N(1) = 1$. The characteristic function has form [6]

$$K_N(x) = x^M \Big(\frac{x_0^2 - x^2}{1 - x_0^2 x^2} \Big)^L \tag{5}$$

where $N = M + 2L$ is the filter degree and x_0 is zero of transfer function and real number ($x_0 < 1$). If x is continuous time angular frequency i.e. $x = \Omega$, then function (4) is the magnitude characteristic of the continuous-time lowpass transfer function. On the other hand, if $x = \tan(\omega/2)/\tan(\omega_p/2)$ where ω is digital angular frequency and ω_p is 3 dB passband edge, then function (4) is the magnitude characteristic of the discrete-time lowpass transfer function. Thus, frequency variable for the lowpass halfband filter approximation is

$$x^2 = -\frac{(z-1)^2}{(z+1)^2} \tag{6}$$

with $z = e^{j\omega}$ and $\tan(\omega_p/2) = 1$, since $\omega_p = \pi/2$ for the halfband filter.

Last step is calculation of poles and zeros in the z plane of the squared magnitude function (4) using Equation (6), which gives digital squared-magnitude function with all poles on the imaginary axis. By adoption poles inside the unit circle, digital lowpass transfer function, $H_0(z)$, is obtained. Using complementary decomposition, this transfer function is separated into two allpass filters.

III. RNS Based IIR Filter

The Residue Number System has possibility to provide high-speed efficient implementation if the structures, used as a basis for RNS implementation, have only operations of addition, multiplication and scaling. Since in the residue number system, addition, subtraction and multiplication are performed with extended precision, the only errors occur are those during the scaling process. Such structures will also exhibit low sensitivity to quantization noise. Therefore, the direct and canonic forms of recursive filter will be considered.

Let $x_i = \langle X \rangle_{m_i}$ denote remainder of X when divided by m_i, $i = 1, 2, 3$. For given set of co-prime numbers $\mathcal{B} = \{m_1, m_2, m_3\}$, every integer X in range $M = \prod_{i=0}^{3} m_i$ have unique representation $X \xrightarrow{\mathcal{B}} (x_1, x_2, x_3)$. The set $\mathcal{B}$ is called the RNS base. This paper investigates an RNS IIR filter bank for the special moduli set $\{2^n - 1, 2^n, 2^n + 1\}$.

To represent negative numbers, the dynamic range is divided into two equal parts, so that any integer inside the range $[-(M-1)/2, (M-1)/2]$ for M odd, or in $[-M/2, M/2-1]$ for M even, can be represented uniquely in RNS. If RNS representation of number X is (x_1, x_2, x_3), then the RNS representation of the complement of X is $(\langle m_1 - x_1 \rangle_{m_1}, \langle m_2 - x_2 \rangle_{m_2}, \langle m_3 - x_3 \rangle_{m_3})$.

The reconstruction of X from (x_1, x_2, x_3) is based on the Chinese Remainder Theorem (CRT)

$$X = \Big\langle \sum_{i=1}^{3} M_i \langle M_i^{-1} \rangle_{m_i} \Big\rangle_M, \tag{7}$$

where $M_i = M/m_i$ and $\langle M_i^{-1} \rangle_{m_i}$ is the multiplicative inverse of M_i modulo m_i, such that $\langle M_i M_i^{-1} \rangle_{m_i} = 1$.

All arithmetic operations over two integers in binary number system, in the residue number system are performed as modulo arithmetic operations over the residues. Consider two binary numbers X, Y and their's corresponding RNS representations $X = (x_1, x_2, x_3)$ and $Y = (y_1, y_2, y_3)$. If $Z = X \ op\ Y$, (op represents one of the arithmetic operations of addition, subtraction, or multiplication) then $Z = (z_1, z_2, z_3)$ in RNS, where $z_i = \langle x_i \ op\ y_i \rangle_{m_i}$, for $i = 1, 2$ and 3.

A. Architecture of Filter

The N degree RNS coded recursive filter consists of three parallel channels, operating in each modulus m_i, with one common scaling array [2].

The below Figure 2 shows RNS based IIR filter implementation in direct form I [3]. The $X(n)$ and $Y(n)$ are the input and output of this filter. The forward converter, scaler which includes reverse conversion (based on the special moduli set and the Chinese Remainder Theorem CRT) and the three IIR filter channels, are used here to speed up the process.

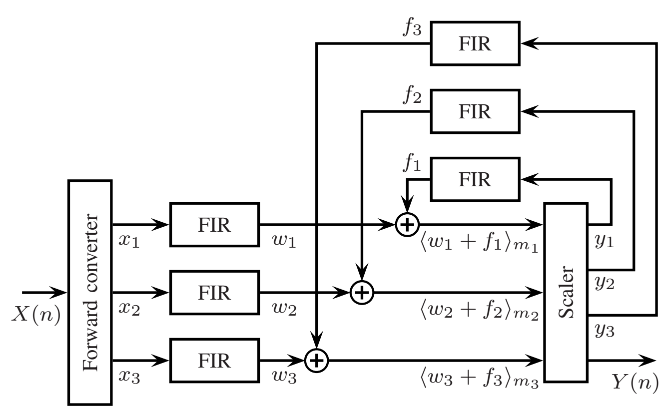

Figure 2. RNS implementation of IIR filter based on three moduli set using FIR sections and RNS scaler.

In order to implement this filter using the RNS arithmetic, the transfer function coefficients and input signal have to be coded by converting the system of floating point variables into integers. It is performed by multiplying them by an appropriate conversion factor, K, and rounding the result to the nearest integer, hence:

$$y_i(i) = \Big\langle \frac{1}{K}\Big\{ \sum_{j=0}^{N} \langle B_j\rangle_{m_i} \langle x(i-j)\rangle_{m_i} + \sum_{j=1}^{N} \langle -A_j\rangle_{m_i} \langle y(i-j)\rangle_{m_i} \Big\} \Big\rangle_{m_i} \tag{8}$$

where B_j and A_j are scaled up forward and reverse coefficients, respectively. Implementation of FIR filters does not require scaling [5], but for the implementation of IIR filters scaling is necessary.

1) RNS Scaling Technique: The RNS scaling technique is based on the special moduli set. According to the Chinese Reminder Theorem (7) it follows

$$Y = \Big\lfloor \Big\langle m_3 \ \langle M_1^{-1} \rangle_{m_1} \ x_1 + \frac{m_1 m_3}{m_2} \ \langle M_2^{-1}\rangle_{m_2} \ x_2 + m_1 \ \langle M_3^{-1}\rangle_{m_3} \ x_3 \Big\rangle_{m_1 m_3} \Big\rfloor \tag{9}$$

where K is chosen to be equal to m_2. The multiplicative inverses of this moduli set are given in [3]. Since $y_i = \langle Y\rangle_{m_i}$ and $\langle\langle X\rangle_{m_1 m_2}\rangle_{m_i} = \langle X\rangle_{m_i}$ then

$$y_i = \Big\langle \Big\langle m_3 \ \langle M_1^{-1} \rangle_{m_1} \ x_1 + \frac{m_1 m_3}{m_2} \ \langle M_2^{-1}\rangle_{m_2} \ x_2 + m_1 \ \langle M_3^{-1}\rangle_{m_3} \ x_3 \Big\rangle_{m_1 m_3} \Big\rangle_{m_i}. \tag{10}$$

By independently computing each of the residue in (10) for $i = 1$, it can be reduced to

$$y_1 = \big\langle x_2 - x_1 \big\rangle_{2^n+1} \tag{11}$$

because $2^{2n} - 2^n - 1 \gg 2^{-n}$. Since for $i = 3$ we have $y_3 = \langle Y\rangle_{m_3}$ and $\langle\langle X\rangle_{m_1 m_3}\rangle_{m_3} = \langle X\rangle_{m_3}$ then

$$y_3 = \big\langle x_3 - x_2 \big\rangle_{2^n-1}. \tag{12}$$

The residue decoder based on the CRT is used to compute scaled output as unsigned binary number [3]

$$Y = \langle A + B + C\rangle_{2^{2n}-1} \tag{13}$$

where $C = \langle(-2^{2n-1} + 2^{n-1})\ x_1\rangle_{2^{2n}-1}$, $B = \langle -2^n \ x_2\rangle_{2^{2n}-1}$ and $A = \langle(2^{2n-1} + 2^{n-1})\ x_3\rangle_{2^{2n}-1}$.

The dynamic range of scaled output is $M_{sc} = (2^{2n} - 1)$. Since system supports signed numbers then dynamic range is $[-M_{sc}/2, M_{sc}/2 - 1]$ for M_{sc} odd. Thus

$$Y = \begin{cases} \langle A + B + C\rangle_{2^{2n}-1} & \text{for } Y < M_{sc}/2 \\ -M_{sc} + \langle A + B + C\rangle_{2^{2n}-1} & \text{for } Y \geq M_{sc}/2 \end{cases} \tag{14}$$

where Y is an integer within the range $0 \leq Y < M_{sc}$. Finally, generator modulo 2^n is used to compute binary number y_2, i.e. to compute $y_2 = \langle Y\rangle_{2^n}$.

The resultant residue digits, $Y = (y_1, y_2, y_3)$, of the scaled output $Y(n)$, for all modulo channels are identical to the scaled integer output with a scaling error that is less than one.

IV. Design Examples

Design parameters are: filter degree $N = 11$, RNS base $\mathcal{B} = \{1023, 1024, 1025\}$ and the scaling factor $K = 2^{10}$. Consider the eleventh order of the proposed lowpass IIR filter

$$H_0(z) = \frac{0.0302(z^{-1}+1)(z^{-2} + 0.6425\ z^{-1} + 1)^5}{1 + 2.657\ z^{-2} + 2.632\ z^{-4} + 1.189\ z^{-6} + 0.2363\ z^{-8} + 0.01563\ z^{-10}}. \tag{15}$$

Then $H_0(z)$ and $H_1(z)$ can be expressed as given in equation (2), where $A_1(z)$ and $A_2(z)$ are allpass functions with stable real integer coefficients

$$A_1(z) = \frac{61 + 692z^{-2} + 1617z^{-4} + 1024z^{-6}}{1024 + 1617z^{-2} + 692z^{-4} + 61z^{-6}},$$
$$A_2(z) = z^{-1}\frac{259 + 1103z^{-2} + 1024z^{-4}}{1024 + 1103z^{-2} + 259z^{-4}}.$$

These allpass functions can be expressed in the following RNS forms:

$$A_{11}(z) = \frac{61 + 692z^{-2} + 594z^{-4} + z^{-6}}{1 + 594z^{-2} + 692z^{-4} + 61z^{-6}}, \tag{16}$$
$$A_{12}(z) = \frac{61 + 692z^{-2} + 593z^{-4}}{593z^{-2} + 692z^{-4} + 61z^{-6}}, \tag{17}$$
$$A_{13}(z) = \frac{61 + 692z^{-2} + 592z^{-4} + 1024z^{-6}}{1024 + 592z^{-2} + 692z^{-4} + 61z^{-6}}, \tag{18}$$

and

$$A_{21}(z) = \frac{259z^{-1} + 80z^{-3} + z^{-5}}{1 + 80z^{-2} + 259z^{-4}}, \tag{19}$$
$$A_{22}(z) = \frac{259 + 79z^{-2}}{79z^{-1} + 259z^{-3}}, \tag{20}$$
$$A_{23}(z) = \frac{259z^{-1} + 78z^{-3} + 1024z^{-5}}{1024 + 78z^{-2} + 259z^{-4}}. \tag{21}$$

Figure 3 shows the impulse responses of two allpass filters and the lowpass filter. Impulse response of the lowpass filter is the sum of impulse responses of two allpass filters.

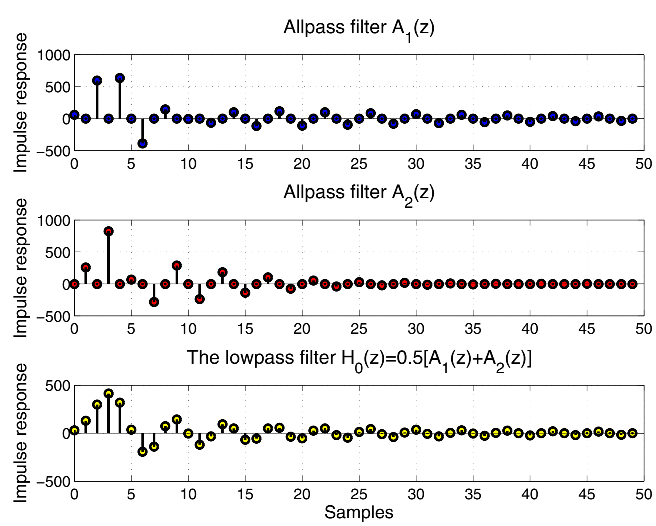

Figure 3. Impulse responses for the allpass filters and the lowpass filter.

It is known that IIR digital filters belong to the class of linear time invariant systems, characterized by the properties of causality, stability and recursivity. They can be characterized in the time domain by their unit-impulse response and in the frequency domain by their transfer function.The FFT can be used to perform a Fourier transform of the impulse response and to compute the frequency response of the filter. The impulse response and the frequency response create the Fourier transform pair $h(t) \leftrightarrow H(f)$, where $h(t)$ is the impulse response and $H(f)$ is the frequency response.

In Figs. 3 and 4 are shown the impulse and the frequency response. The number of sample points used for FFT evaluation is equal to $N_{fft} = 500$. Amplitude characteristic is modulus of FFT. For standard implementation, filter coefficients are rounded to 10 bits, but arithmetic operations are rounded to 16 bits. Base for RNS implementation is $\mathcal{B} = \{1023,\ 1024,\ 1025\}$.

Relation that connects the frequency resolution and the total acquisition time, is relation between the sampling frequency and the block size of the FFT, and is given by the following formula $\Delta f = 1/T = F_s/N_{fft}$, where Δf is the frequency resolution, T is the acquisition time, F_s is the sampling frequency, and N_{fft} is the block size of the FFT.

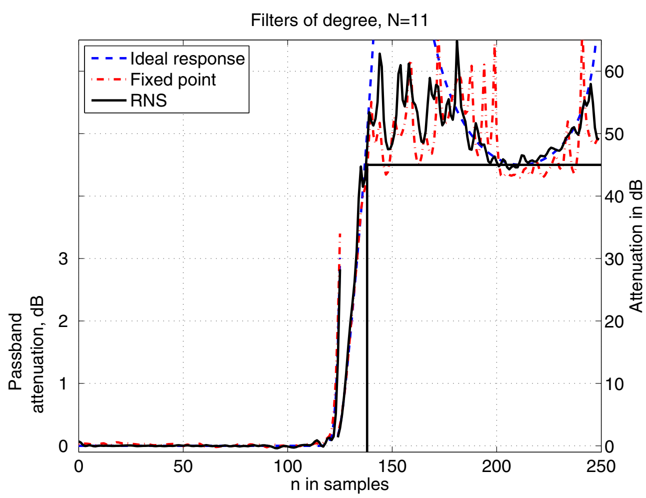

Figure 4. Magnitude responses of the 11th degree lowpass filter calculated as FFT of its lowpass impulse response shown in Figure 3.

Magnitude responses corresponding to the ideal impulse response in two's complement fixed point arithmetic, are also presented in Figure 4 for comparison. It can be shown that in two's complement arithmetic, choice of 10-bits precision for coefficients and 19-bits precision for arithmetic operations, has not satisfied frequency requirements. Only choice of more than 12-bits for coefficient precision and kipping 19-bits, meets the frequency requirements.

Important properties of the final implementation of the signal processor are speed, power and small size. There are mainly three parameters in digital filters that affect those properties: filter degree, filter coefficient length and data word length.

Data word length is 19 bits including the sign bit, and it is transformed by RNS representation into three 10-bits residue digits. All arithmetic operations performed in modulo m_i are accurate without rounding errors. Thus, in case of truncation scaling, errors that occur during the scaling process can be at most unity.

In two's complement fixed arithmetic, multiplication of two numbers that are 12-bits (filter coefficient) and 19-bits (data word) length, yields to 31-bits result which should be truncated to 19 bits. This error is much greater in comparison to RNS arithmetic.

V. Conclusions

The arithmetic operations of addition, subtraction, and multiplication can be speeded up due to the parallel processing properties of the residue number system. However, some difficult operations also need further investigation in order to reduce the complexity. The operation of scaling is typical operation difficult to be performed. In this paper, presented algorithm for the 2^n scaling is based on Chinese remainder theorem (CRT).

Furthermore, simulation of the two channel IIR quadrature mirror filter bank, based on residue arithmetic, is presented. The RNS implementation is more efficient, regarding the arithmetic operation performing and rounding error, than its corresponding conventional implementation.

In comparison with standard realization, RNS realization has following advantages:

- Arithmetic operations are with unsigned integer numbers.
- Arithmetic operations are without rounding errors, except the scaling error that is less than one.
- Data word length is much smaller.

Further research should be related to the implementation of the scaling operation by an arbitrary scaling factor, and the RNS scaler for a four moduli set.

Acknowledgment

This work is supported by Serbian Ministry of Science and Technologies, Project No. 32009TR.

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